AP Physics C Review
Mechanics

CHSN Review Project

This is a review guide designed as preparatory information for the AP[1] Physics C Mechanics Exam on May 11, 2009. It may still, however, be useful for other purposes as well. Use at your own risk. I hope you find this resource helpful. Enjoy!

This review guide was written by Dara Adib based on inspiration from Shelun Tsai's review packet.

This is a development version of the text that should be considered a work-in-progress.

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"Why do we love ideal worlds? ... I've been doing this for 38 years and school is an ideal world." — Steven Henning

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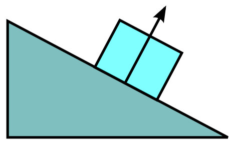

Figure 1: Normal Force

Kinematic Equations

$$\Delta x = \frac{1}{2}at^2 + v_0 t$$

$$\Delta v = at$$

$$(v)^2 - (v_0)^2 = 2a(\Delta x)$$

$$\Delta x = \frac{v_0 + v}{2} \times t$$

Free Body Diagrams

N Normal Force

f Frictional Force

T Tension

mg Weight

$$F = ma$$

In a particular direction:

$$\Sigma F = (\Sigma m)a$$

Atwood's Machine[2]

$$a = \frac{|(m_2 - m_1)|g}{m_1 + m_2}$$

[2] Pulley and string are assumed to be massless.

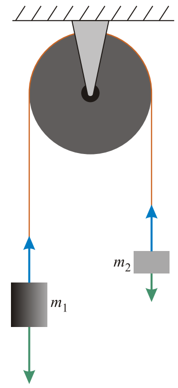

Figure 2: Atwood's Machine

Figure 3: Draw a banked curve diagram

Pulled Weights

$$a = \frac{F - f}{\Sigma m}$$

$$T = ma$$

Elevator

Normal force acts upward, weight acts downward.

- Accelerating upward: $N = |ma| + |mg|$
- Constant velocity: $N = |mg|$
- Accelerating downward: $N = |mg| - |ma|$

Banked Curve

Friction can act up the ramp (minimum velocity when friction is maximum) or down the ramp (maximum velocity when friction is maximum).

$$v_{\text{ideal}} = \sqrt{rg \tan \theta}$$

$$v_{min} = \sqrt{\frac{rg(\tan\theta - \mu)}{\mu\tan\theta + 1}}$$

$$v_{max} = \sqrt{\frac{rg(\tan\theta + \mu)}{1 - \mu\tan\theta}}$$

Projectile Motion

Position

$$\Delta x = v_x t$$

$$\Delta y = -\frac{1}{2}gt^2 + (v_y)_0 t$$

Velocity

θ represents the smaller angle from the x-axis to the direction of the projectile's initial motion.

$$(v_x)_0 = v_0 \cos\theta$$

$$(v_y)_0 = v_0 \sin\theta$$

$$\Delta v_x = 0$$

$$\Delta v_y = -gt$$

Height

θ represents the smaller angle from the x-axis to the direction of the projectile's initial motion.

Starting from a height of $x = 0$:

$$y_{max} = \frac{(v_0 \sin\theta)^2}{2g}$$

Range

θ represents the smaller angle from the x-axis to the direction of the projectile's initial motion.

Starting from a height of $x = 0$:

$$x_{max} = \frac{(v_0)^2 \sin 2\theta}{g}$$

Circular Motion

Centripetal (radial)

Centripetal acceleration and force is directed towards the center. It refers to a change in direction.

$$a_c = \frac{v^2}{r}$$

$$F_c = ma_c = \frac{mv^2}{r}$$

Tangential

Tangential acceleration is tangent to the object's motion. It refers to a change in speed.

$$a_t = \frac{d|v|}{dt}$$

Combined

$$a_{total} = \sqrt{(a_c)^2 + (a_t)^2}$$

Vertical loop

In a vertical loop, the centripetal acceleration is caused by a normal force and gravity (weight).

Top

$$\begin{aligned} F &= ma \\ N + mg &= m \times \frac{v^2}{r} \\ N &= \frac{mv^2}{r} - mg \end{aligned}$$

Bottom

$$\begin{aligned} F &= ma \\ N - mg &= m \times \frac{v^2}{r} \\ N &= \frac{mv^2}{r} + mg \end{aligned}$$

Friction

Friction converts mechanical energy into heat. Static friction (at rest) is generally greater than kinematic friction (in motion).

$$f_{max} = \mu N$$

Momentum-Impulse

$$p = mv$$

$$F = \frac{dp}{dt}$$

$$I = \int F dt = \bar{F}\Delta t = \Delta p = m\Delta v$$

Collisions

Total momentum is always conserved when there are no external forces ($F = \frac{dp}{dt} = 0$).

Elastic

Kinetic energy is conserved.

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$$

$$-(v_2' - v_1') = v_2 - v_1$$

Inelastic

Kinetic energy is not conserved.

$$m_1 v_1 + m_2 v_2 = (m_1 + m_2) v'$$

Center of Mass

$$r_{cm} = \frac{\Sigma m r}{\Sigma m} = \frac{1}{\Sigma m} \int r dm = \frac{1}{\Sigma m} \int x \lambda dx$$

$$\lambda = \frac{dm}{dx} = \frac{M}{L}$$

$$\Sigma m = \int dm = \int \lambda dx$$

$$(\Sigma m) v_{CM} = \Sigma m v = \Sigma p$$

$$F_{net} = (\Sigma m) a_{CM}$$

Energy

Work

$$W = \int F dx = \Delta K$$

Power

$$P_{avg} = \frac{W}{t} = \frac{Fx}{t}$$

$$P_{instant} = \frac{dW}{dt} = Fv$$

Kinetic Energy

$$K = \frac{1}{2}mv^2$$

Potential Energy

$$F = -\frac{dU}{dx}$$

$$\Delta U = -\int_{x_i}^{x_f} F_C dx = -W_C$$

$$U_{Hooke} = -\int F_{Hooke} dx = -\int -kx dx = \frac{1}{2}kx^2$$

$$U_g = mgh$$

equilibrium point $F = -\frac{du}{dx} = 0$ (extrema)

stable equilibrium U is a minimum

unstable equilibrium U is a maximum

Total

$$E = K + U$$

$$E_i + W_{NC} = E_f$$

W_{NC} represents non-conservative work that converts mechanical energy into other forms of energy. For example, friction converts mechanical energy into heat.

Linear	Angular
$v = \frac{dx}{dt} = \frac{\Delta x}{\Delta t}$	$\omega = \frac{d\theta}{dt} = \frac{\Delta\theta}{\Delta t}$
$a = \frac{dv}{dt} = \frac{\Delta v}{\Delta t}$	$\alpha = \frac{d\omega}{dt} = \frac{\Delta\omega}{\Delta t}$
$\Delta x = \frac{1}{2}at^2 + v_0 t$	$\Delta\theta = \frac{1}{2}\alpha t^2 + \omega_0 t$
$\Delta v = at$	$\Delta\omega = \alpha t$
$(v)^2 - (v_0)^2 = 2a(\Delta x)$	$(\omega)^2 - (\omega_0)^2 = 2\alpha(\Delta\omega)$
$\Delta x = \frac{v_0 + v}{2} \times t$	$\Delta\theta = \frac{\omega_0 + \omega}{2} \times t$
$F = ma$	$\tau = I\alpha$
$W = \int_{x_0}^{x} F dx$	$W_{rot} = \int_{\theta_0}^{\theta} \tau d\theta$
$W = \frac{1}{2}mv^2 - \frac{1}{2}m(v_0)^2$	$W_{rot} = \frac{1}{2}m\omega^2 - \frac{1}{2}m(\omega_0)^2$
$P = Fv$	$P_{rot} = \tau\omega$
$p = mv$	$L = I\omega$
$F = \frac{dp}{dt}$	$\tau = \frac{dL}{dt}$

Figure 4: Rotational Motion

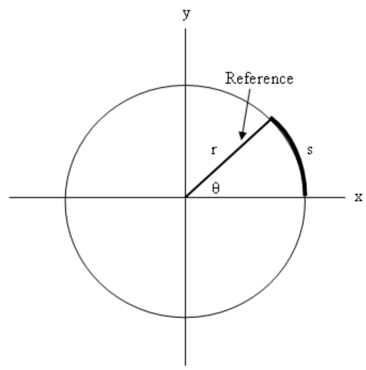

Figure 5: Arc Length

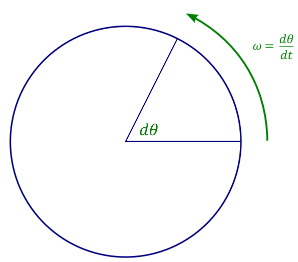

Figure 6: Angular Velocity

Rotational Motion

The same equations for linear motion can be modified for use with rotational motion (Figure 4 on the previous page).

Angular Motion

$$\theta = \frac{s}{r}$$

$$\omega = \frac{v}{r}$$

$$\alpha = \frac{a_t}{r}$$

$$a_t = r\sqrt{\alpha^2 + \omega^4}$$

$$a_c = \omega^2 r$$

$$K_{rolling} = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$$

Torque

$$\tau = r \times F = rF\sin\theta$$

$$\tau = I\alpha$$

Moment of Inertia

$$I = \Sigma mr^2 = \int r^2 dm$$

$$I = I_{cm} + Mh^2$$

(h represents the distance from the center)

Values

rod (center) $\frac{1}{12}ml^2$

rod (end) $\frac{1}{3}ml^2$

hollow hoop/cylinder mr^2

solid disk/cylinder $\frac{1}{2}mr^2$

hollow sphere $\frac{2}{3}mr^2$

solid sphere $\frac{2}{5}mr^2$

Atwood's Machine

$$a = \frac{|(m_2 - m_1)|g}{m_1 + m_2 + \frac{1}{2}M}$$

Angular Momentum

$$L = I\omega$$

$$L = r \times p = rp\sin\theta = rmv\sin\theta$$

$$\tau = \frac{dL}{dt}$$

Total angular momentum is always conserved when there are no external torques ($\tau = \frac{dL}{dt} = 0$).

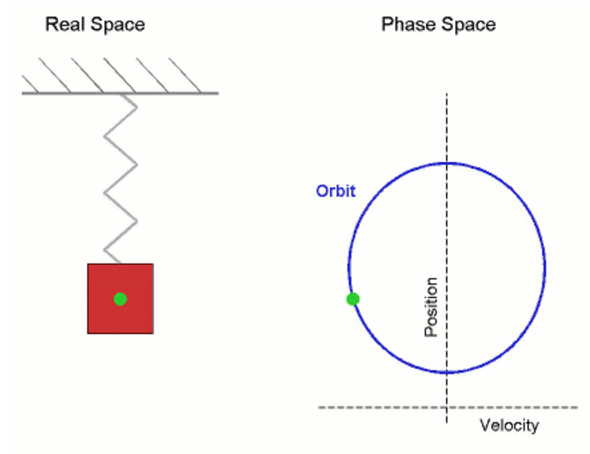

Figure 7: Simple Harmonic Motion

Simple Harmonic Motion

Simple harmonic motion is the projection of uniform circular notion on to a diameter. Likewise, uniform circular motion is the combination of simple harmonic motions along the x-axis and y-axis that differ by a phase of $90°$.

amplitude (A) maximum magnitude of displacement from equilibrium

cycle one complete vibration

period (T) time for one cycle

frequency (f) cycles per time

angular frequency (ω) radians per time

$$x = A\cos(\omega t + \phi)$$

$$v = -\omega A \sin(\omega t + \phi)$$

$$a = -\omega^2 A \cos(\omega t + \phi) = -\omega^2 x$$

$$T = \frac{2\pi}{\omega} = \frac{1}{f}$$

$$f = \frac{1}{T} = \frac{\omega}{2\pi}$$

$$\omega = 2\pi f$$

$$A = \sqrt{(x_0)^2 + \left(\frac{v_0}{\omega}\right)^2}$$

$$\phi = \arctan\left(\frac{-v_0}{\omega x_0}\right)$$

$$E = \frac{1}{2}kA^2$$

Spring

$$F_s = -kx$$

$$T_s = 2\pi\sqrt{\frac{m}{k}}$$

$$\omega_s = \sqrt{\frac{k}{m}}$$

Pendulum

Simple

$$T = 2\pi\sqrt{\frac{L}{g}}$$

$$\omega = \sqrt{\frac{g}{L}}$$

Compound

A cable with a moment of inertia swings back and forth. d represents the distance from the pendulum's pivot to its center of mass.

$$T = 2\pi\sqrt{\frac{I}{mgd}}$$

$$\omega = \sqrt{\frac{mgd}{I}}$$

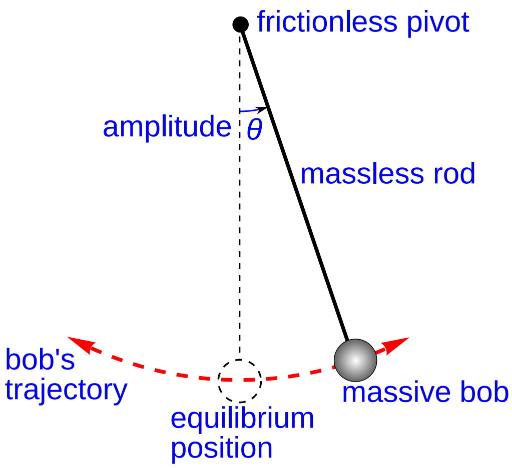

Figure 8: Simple Pendulum

Torsional

A horizontal mass with a moment of inertia is suspended from a cable and swings back and forth.

$$T = 2\pi\sqrt{\frac{I}{k}}$$

$$\omega = \frac{k}{I}$$

Gravity

$$F = \frac{-Gm_1m_2}{R^2}$$

$$G \approx 6.67 \times 10^{-11}\,\frac{Nm^2}{kg^2}$$

Kepler's Laws

1. All orbits are elliptical.
2. Law of Equal Areas.
3. $T^2 = \frac{4\pi^2}{GM}R^3 = K_sR^3$, where K_s is a uniform constant for all satellites/planets orbiting a specific body

Energy

$$U = \frac{-Gm_1m_2}{R}$$

$$E = \frac{-GMm}{2r}$$

$$v = \frac{2\pi R}{T}$$

$$v_{escape} = \sqrt{\frac{2GM}{r_e}}$$

For orbits around the earth, r_e represents the radius of the earth.