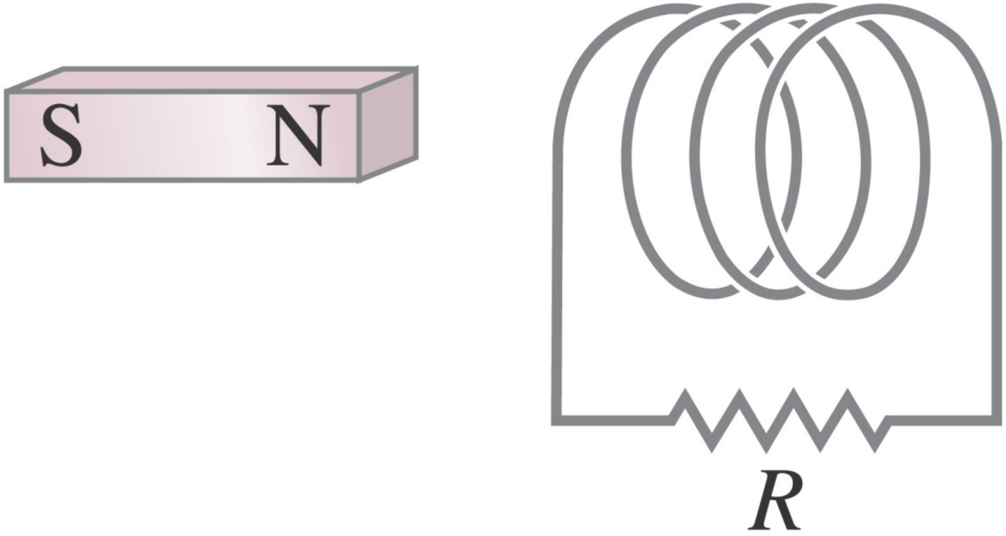
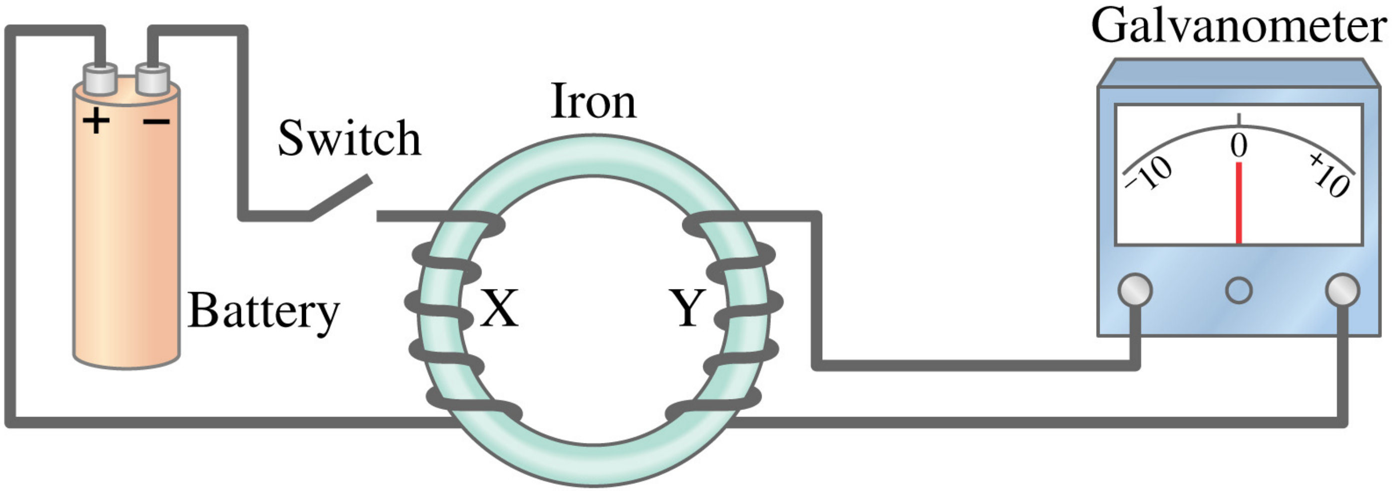
Chapter 21

Electromagnetic Induction and Faraday's Law



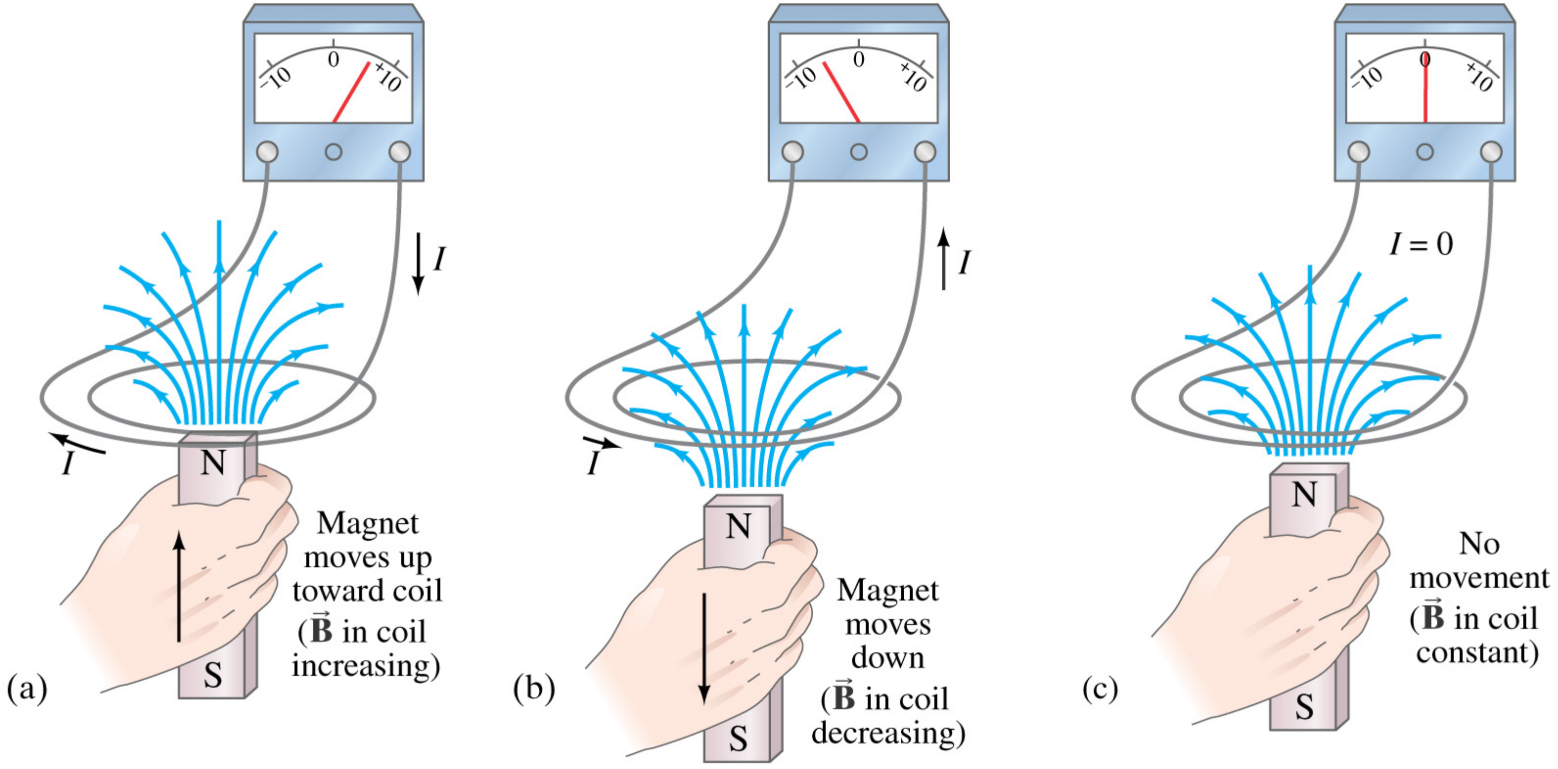
21.1 Induced EMF

Almost 200 years ago, Faraday looked for evidence that a magnetic field would induce an electric current with this apparatus:



21.1 Induced EMF

He found no evidence when the current was steady, but did see a current induced when the switch was turned on or off.



21.1 Induced EMF

Therefore, a changing magnetic field induces an emf.

Faraday’s experiment used a magnetic field that was changing because the current producing it was changing; the previous graphic shows a magnetic field that is changing because the magnet is moving.

21.2 Faraday’s Law of Induction; Lenz’s Law

The induced emf in a wire loop is proportional to the rate of change of magnetic flux through the loop.

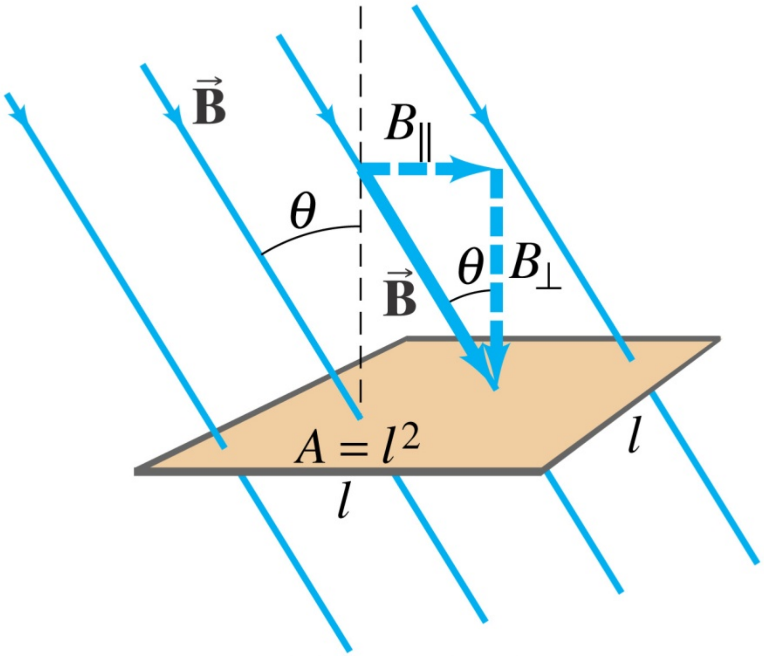
Magnetic flux: $\Phi_B = B_\perp A = BA\cos\theta$ **(21-1)**

Unit of magnetic flux: weber, Wb.

1 Wb = 1 T·m^2

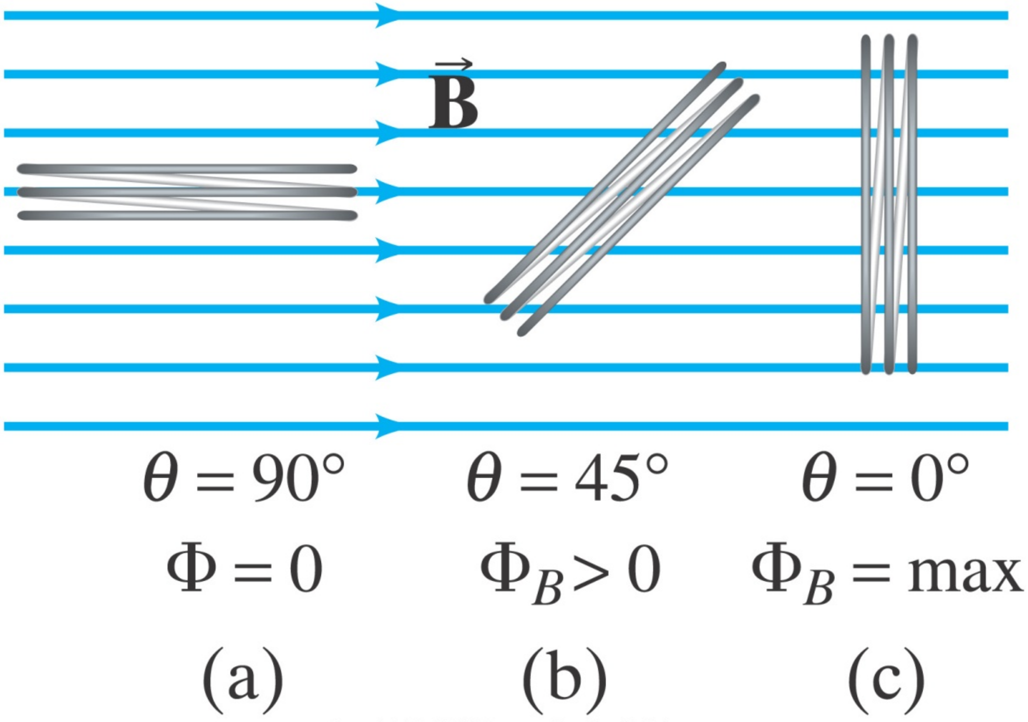
21.2 Faraday’s Law of Induction; Lenz’s Law

This drawing shows the variables in the flux equation:



21.2 Faraday’s Law of Induction; Lenz’s Law

The magnetic flux is analogous to the electric flux – it is proportional to the total number of lines passing through the loop.



21.2 Faraday's Law of Induction; Lenz's Law

Faraday's law of induction:

$$\mathcal{E} = -\frac{\Delta\Phi_B}{\Delta t}$$

[1 loop] (21-2a)

$$\mathcal{E} = -N\frac{\Delta\Phi_B}{\Delta t}$$

[N loops] (21-2b)

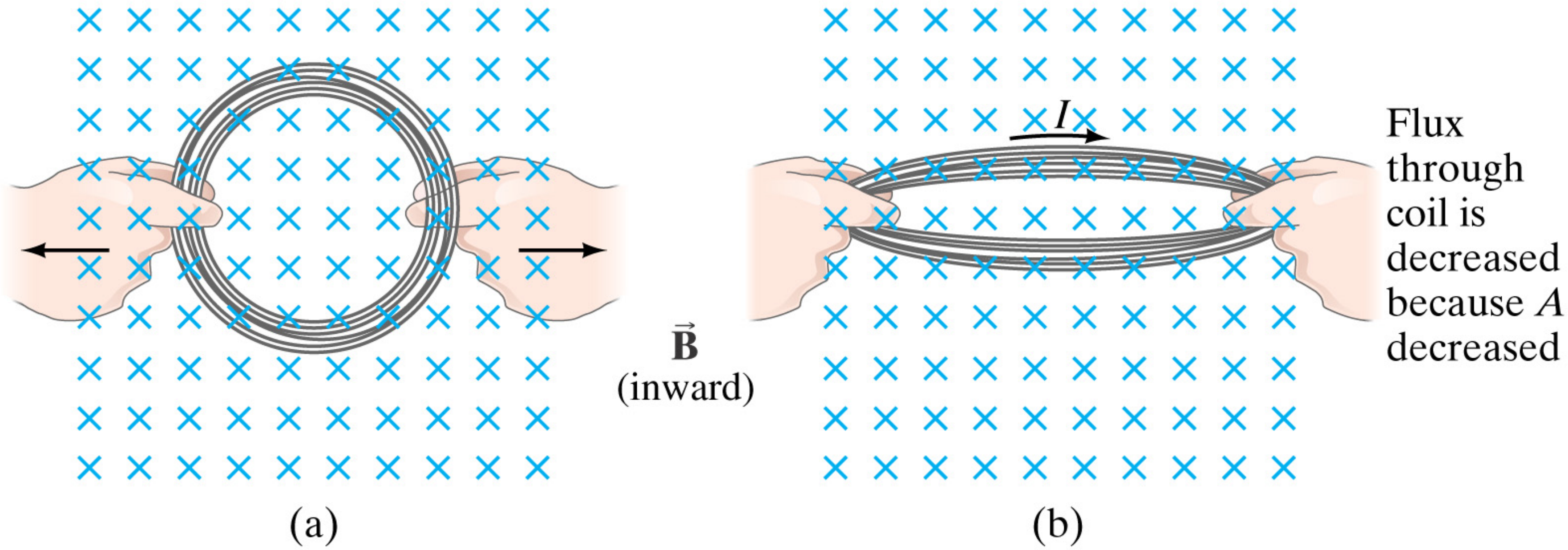
21.2 Faraday’s Law of Induction; Lenz’s Law

The minus sign gives the direction of the induced emf:

A current produced by an induced emf moves in a direction so that the magnetic field it produces tends to restore the changed field.

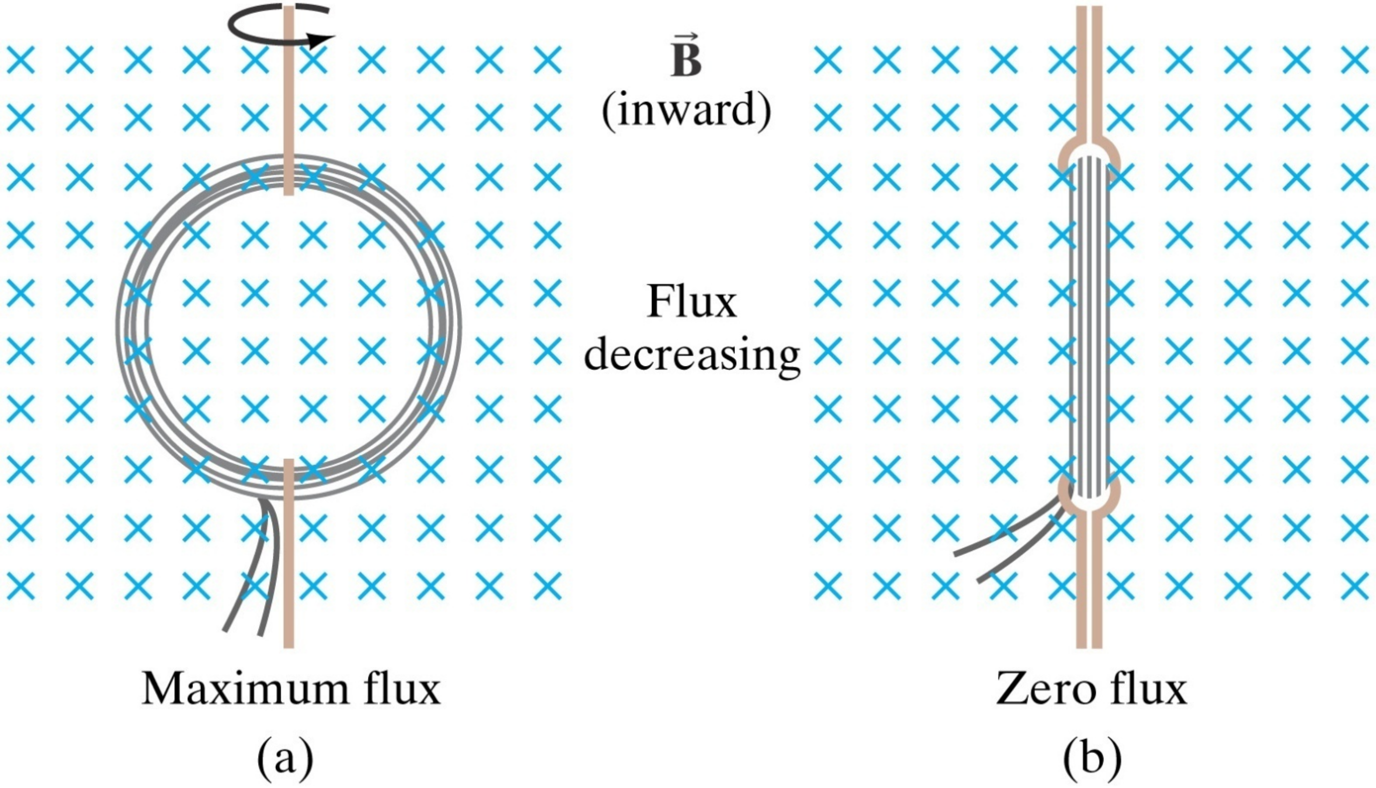
21.2 Faraday's Law of Induction; Lenz's Law

Magnetic flux will change if the area of the loop changes:



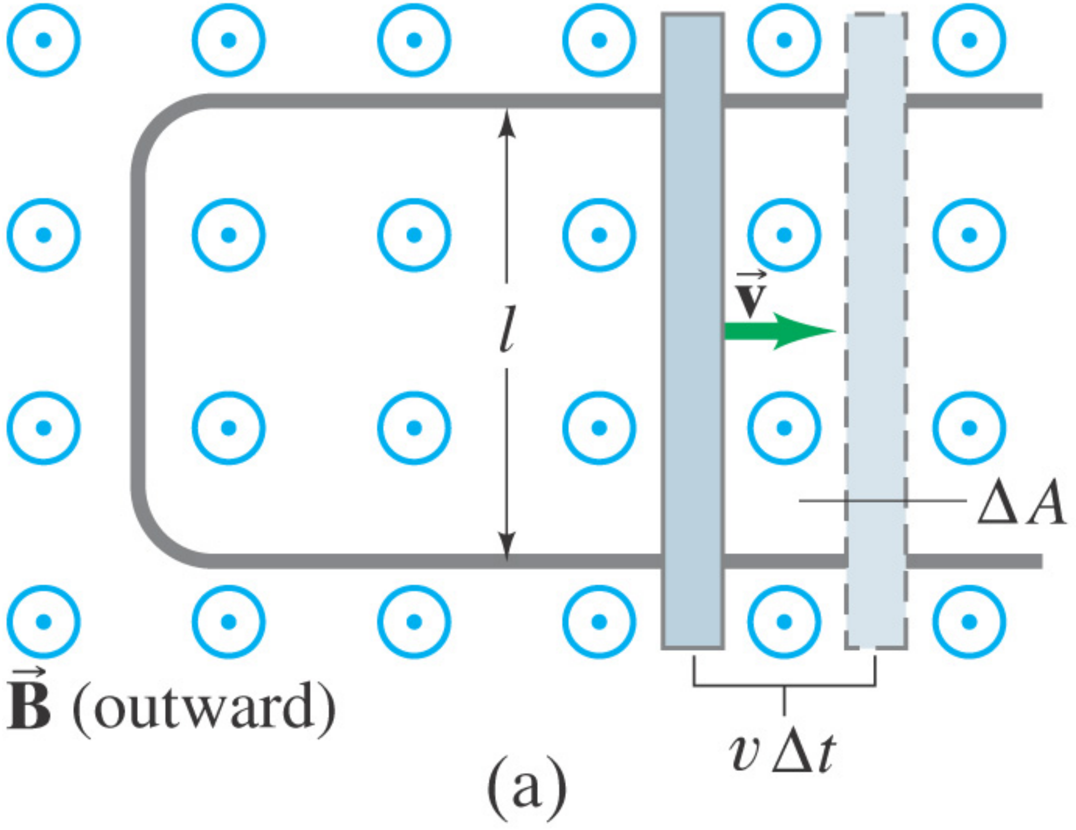
21.2 Faraday's Law of Induction; Lenz's Law

Magnetic flux will change if the angle between the loop and the field changes:



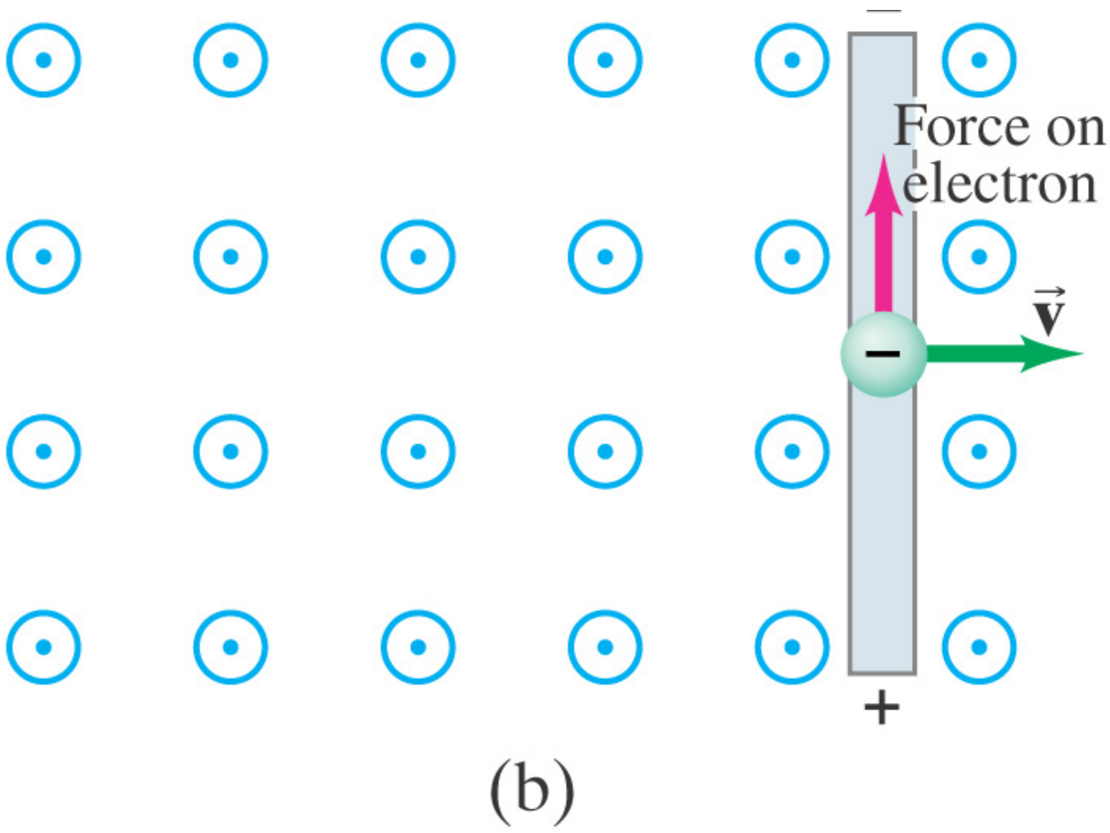
21.3 EMF Induced in a Moving Conductor

This image shows another way the magnetic flux can change:



21.3 EMF Induced in a Moving Conductor

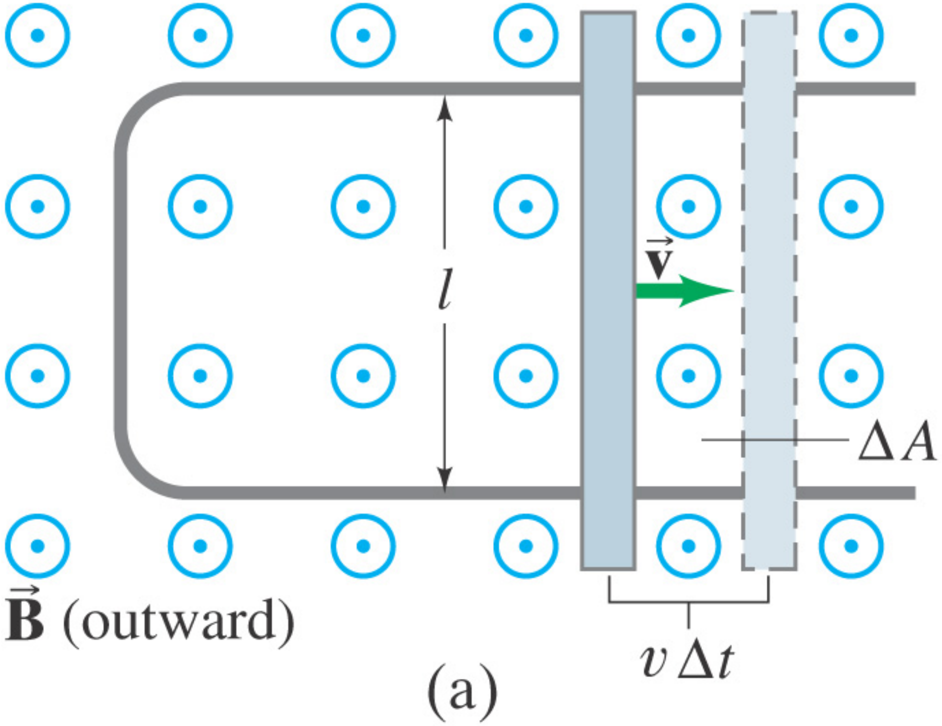
The induced current is in a direction that tends to slow the moving bar – it will take an external force to keep it moving.



21.3 EMF Induced in a Moving Conductor

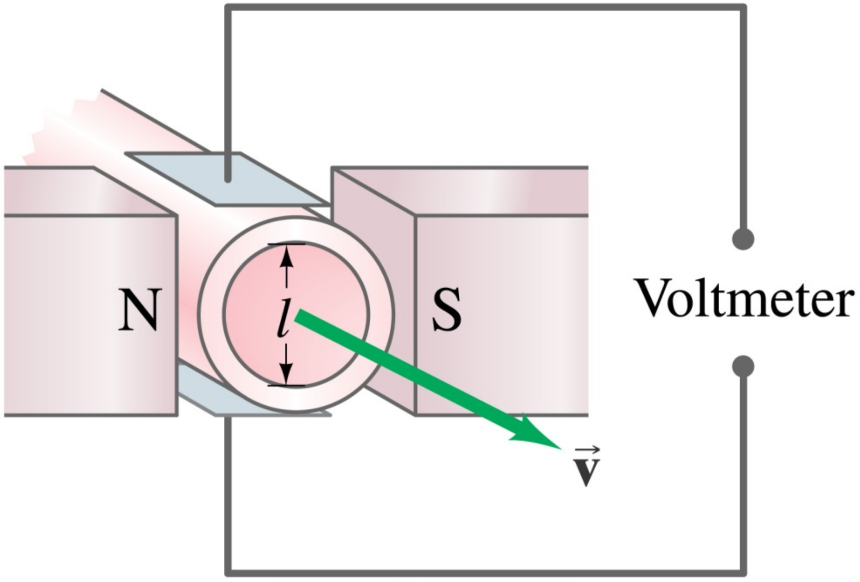
The induced emf has magnitude

$$\mathcal{E} = \frac{\Delta\Phi_B}{\Delta t} = \frac{B\,\Delta A}{\Delta t} = \frac{Blv\,\Delta t}{\Delta t} = Blv \qquad \textbf{(21-3)}$$



Example 21.7

Measurement of blood velocity from induced emf:



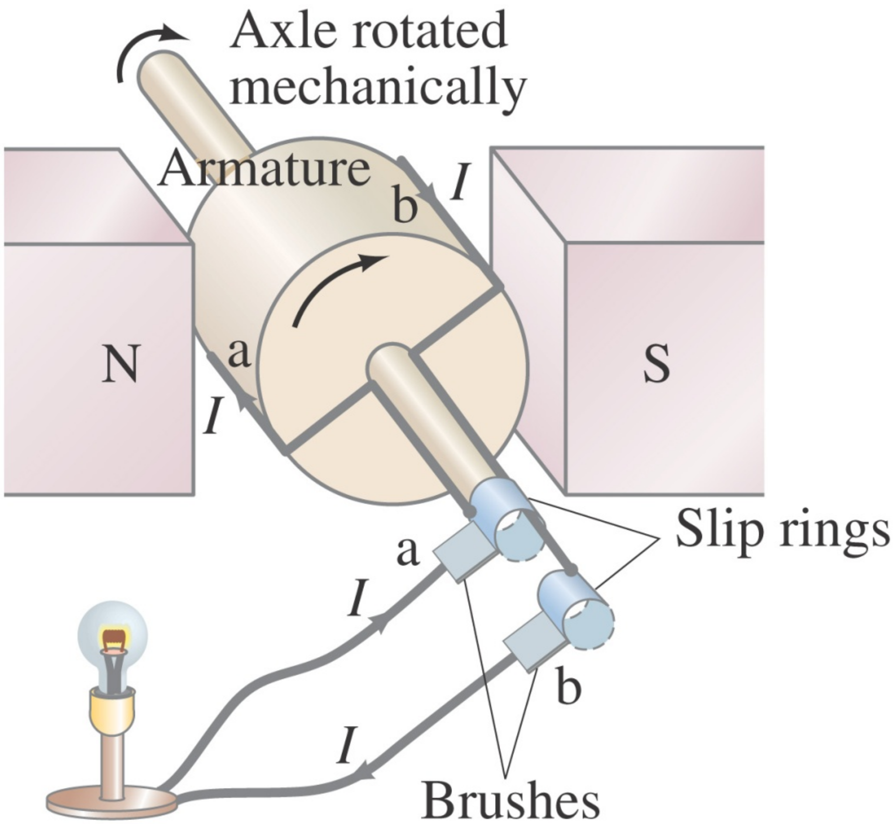
21.4 Changing Magnetic Flux Produces an Electric Field

A changing magnetic flux induces an electric field; this is a generalization of Faraday's law. The electric field will exist regardless of whether there are any conductors around.

21.5 Electric Generators

A generator is the opposite of a motor – it transforms mechanical energy into electrical energy. This is an ac generator:

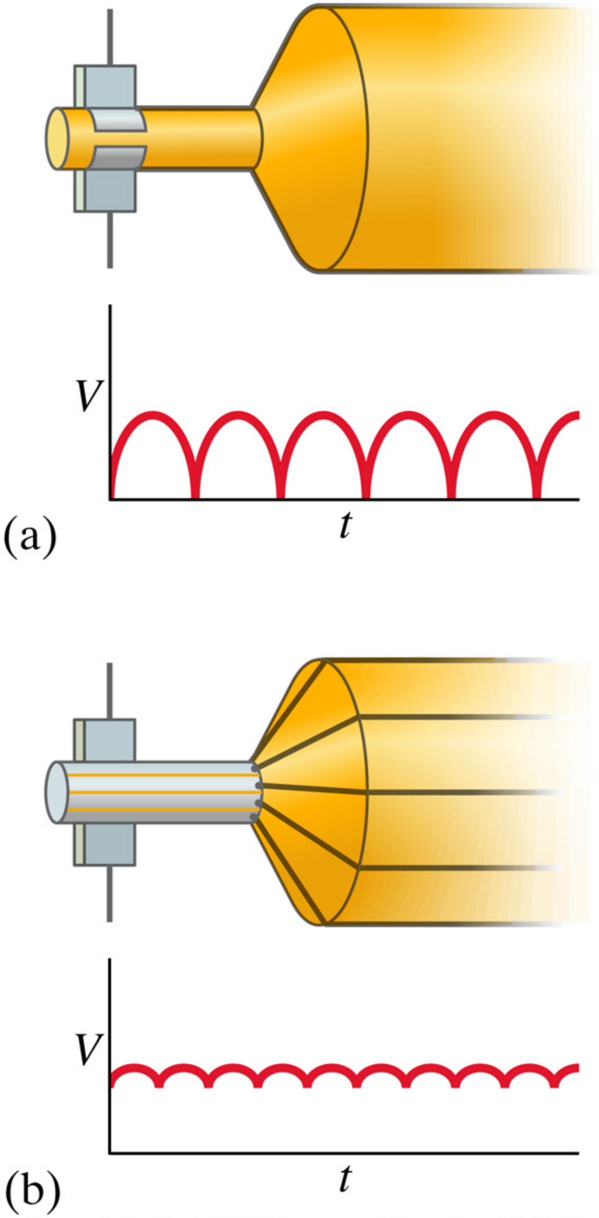
The axle is rotated by an external force such as falling water or steam. The brushes are in constant electrical contact with the slip rings.



Copyright © 2005 Pearson Prentice Hall, Inc.

21.5 Electric Generators

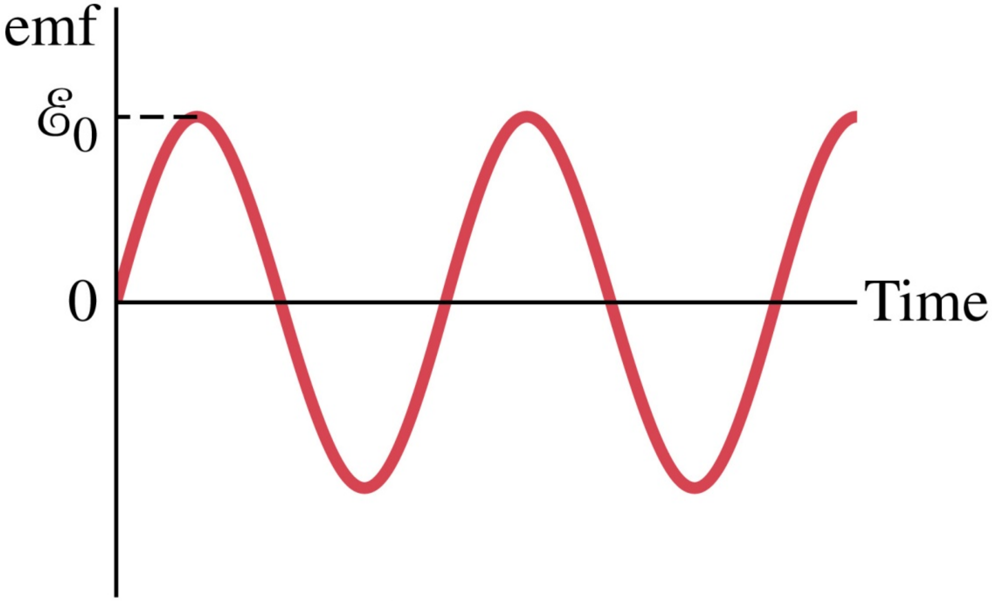
A dc generator is similar, except that it has a split-ring commutator instead of slip rings.



21.5 Electric Generators

A sinusoidal emf is induced in the rotating loop (*N* is the number of turns, and *A* the area of the loop):

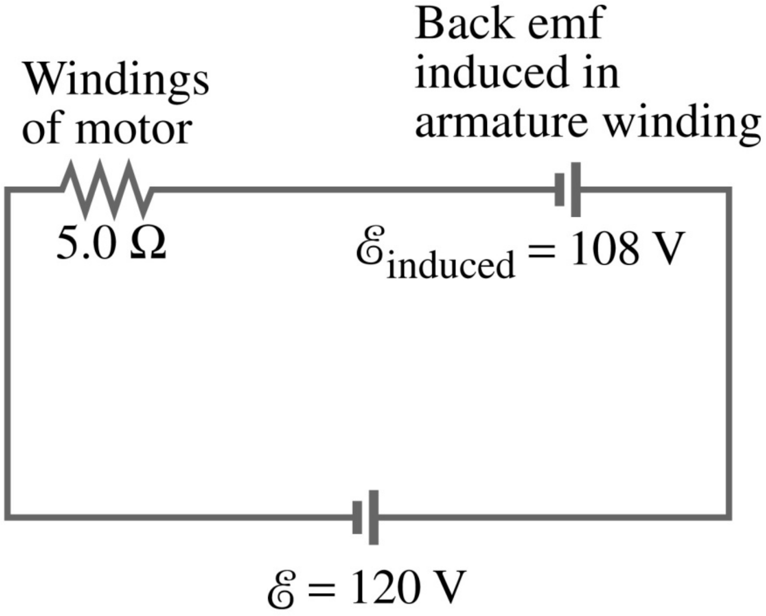
$$\mathcal{E} = NB\omega A \sin \omega t \qquad \textbf{(21-5)}$$



21.6 Back EMF and Counter Torque; Eddy Currents

An electric motor turns because there is a torque on it due to the current. We would expect the motor to accelerate unless there is some sort of drag torque.

That drag torque exists, and is due to the induced emf, called a back emf.

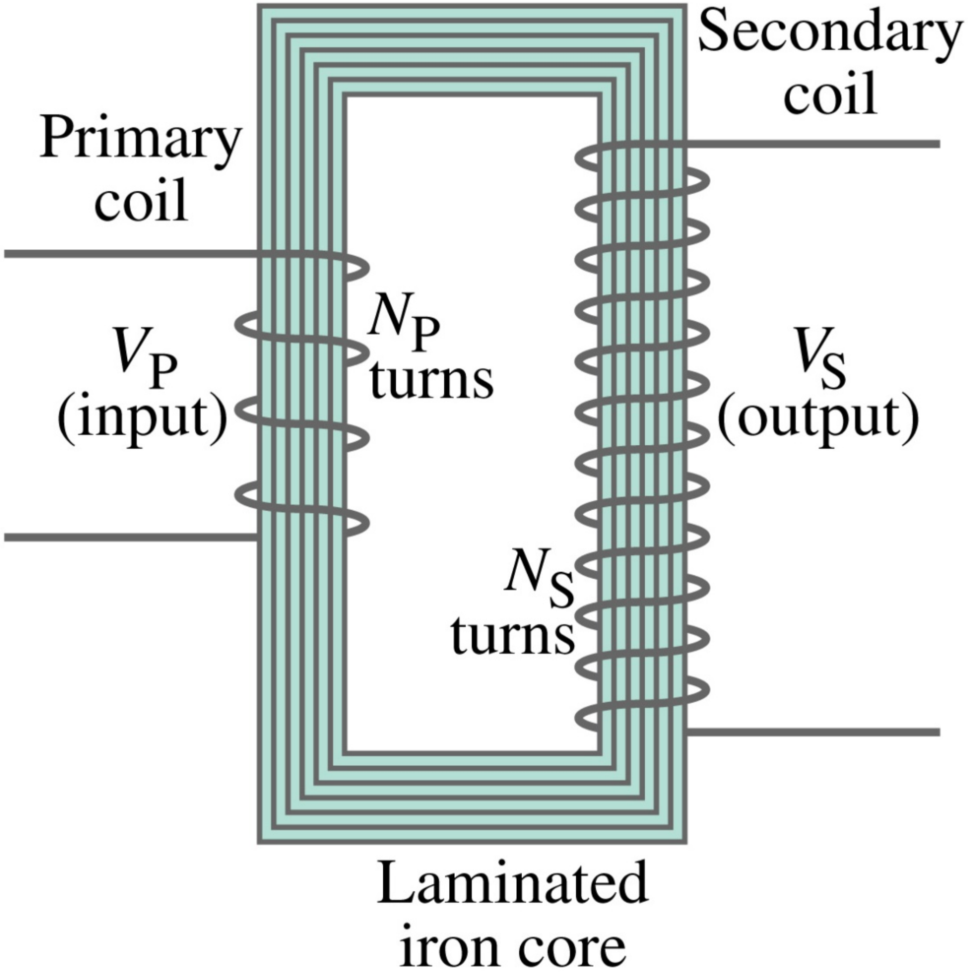


21.6 Back EMF and Counter Torque; Eddy Currents

A similar effect occurs in a generator – if it is connected to a circuit, current will flow in it, and will produce a counter torque. This means the external applied torque must increase to keep the generator turning.

21.7 Transformers and Transmission of Power

This is a step-up transformer – the emf in the secondary coil is larger than the emf in the primary:



21.7 Transformers and Transmission of Power

A transformer consists of two coils, either interwoven or linked by an iron core. A changing emf in one induces an emf in the other.

The ratio of the emfs is equal to the ratio of the number of turns in each coil:

$$\frac{V_S}{V_P} = \frac{N_S}{N_P} \qquad (21\text{-}6)$$

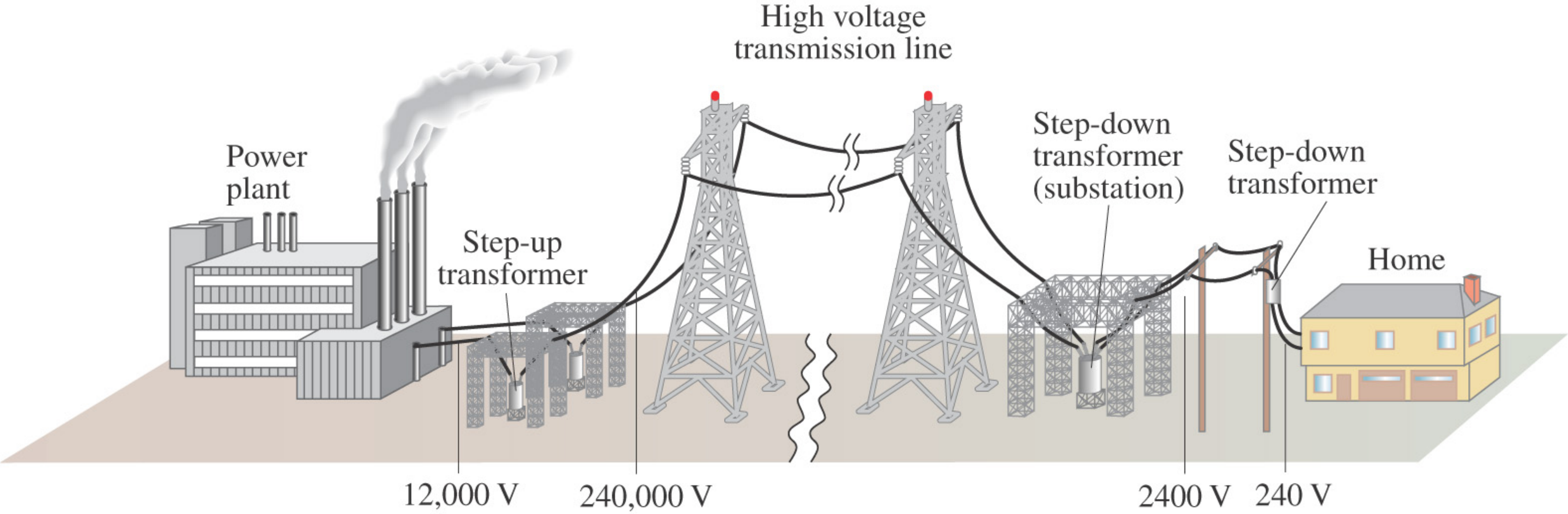
21.7 Transformers and Transmission of Power

Energy must be conserved; therefore, in the absence of losses, the ratio of the currents must be the inverse of the ratio of turns:

$$\frac{I_S}{I_P} = \frac{N_P}{N_S} \qquad (21\text{-}6)$$

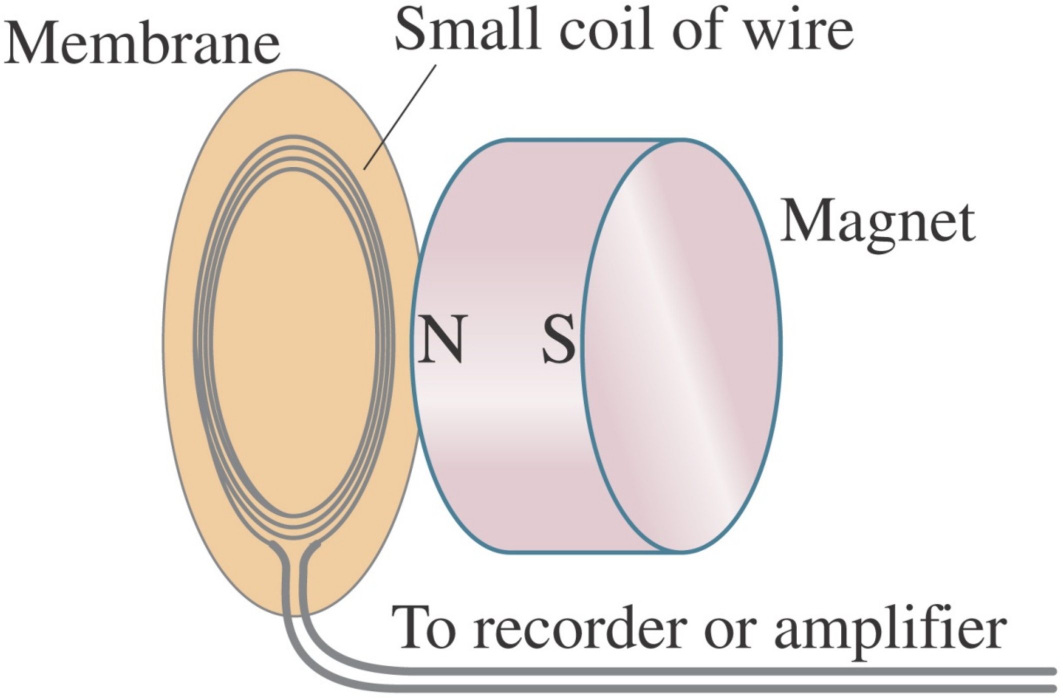
21.7 Transformers and Transmission of Power

Transformers work only if the current is changing; this is one reason why electricity is transmitted as ac.



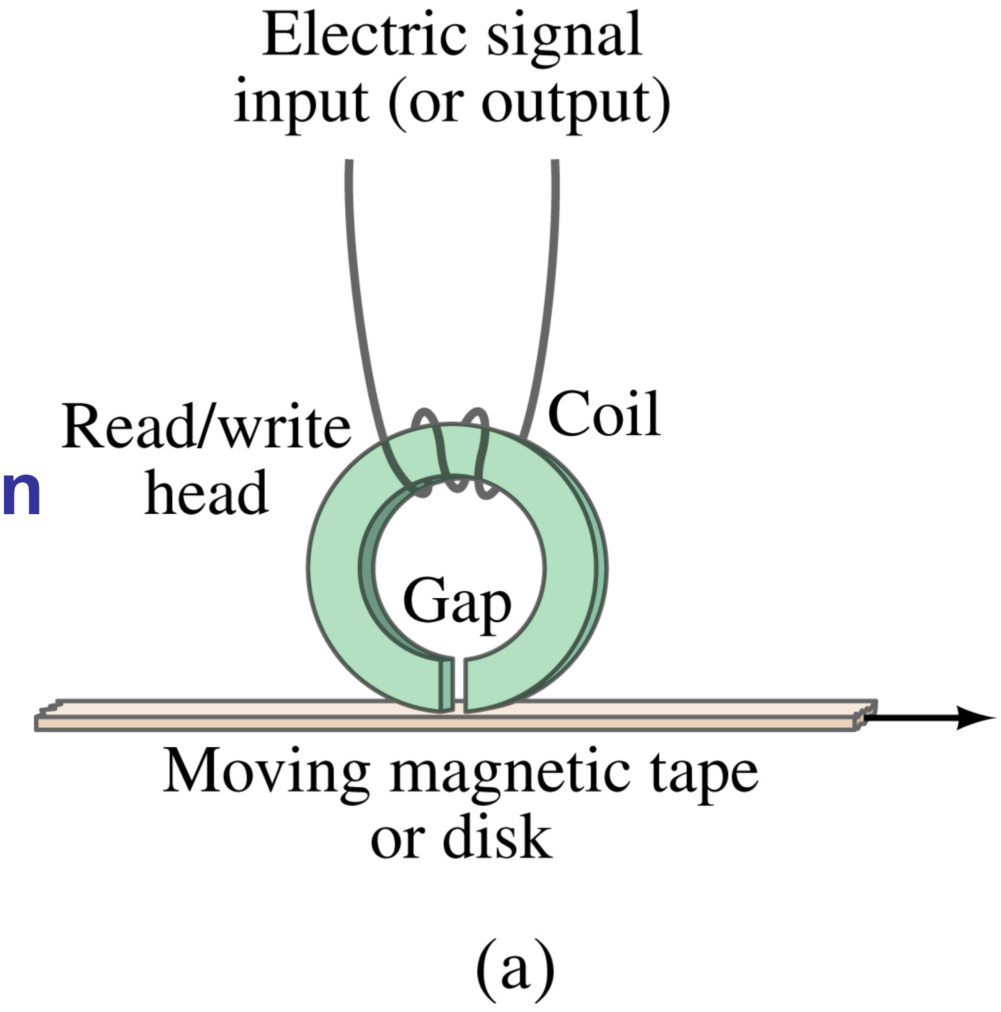
21.8 Applications of Induction: Sound Systems, Computer Memory, Seismograph, GFCI

This microphone works by induction; the vibrating membrane induces an emf in the coil



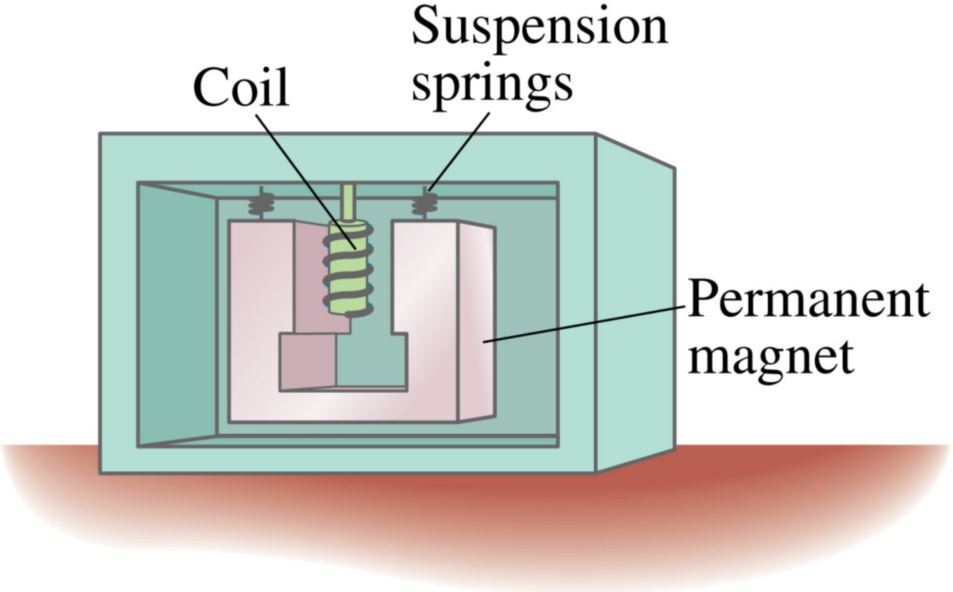
21.8 Applications of Induction: Sound Systems, Computer Memory, Seismograph, GFCI

Differently magnetized areas on an audio tape or disk induce signals in the read/write heads.



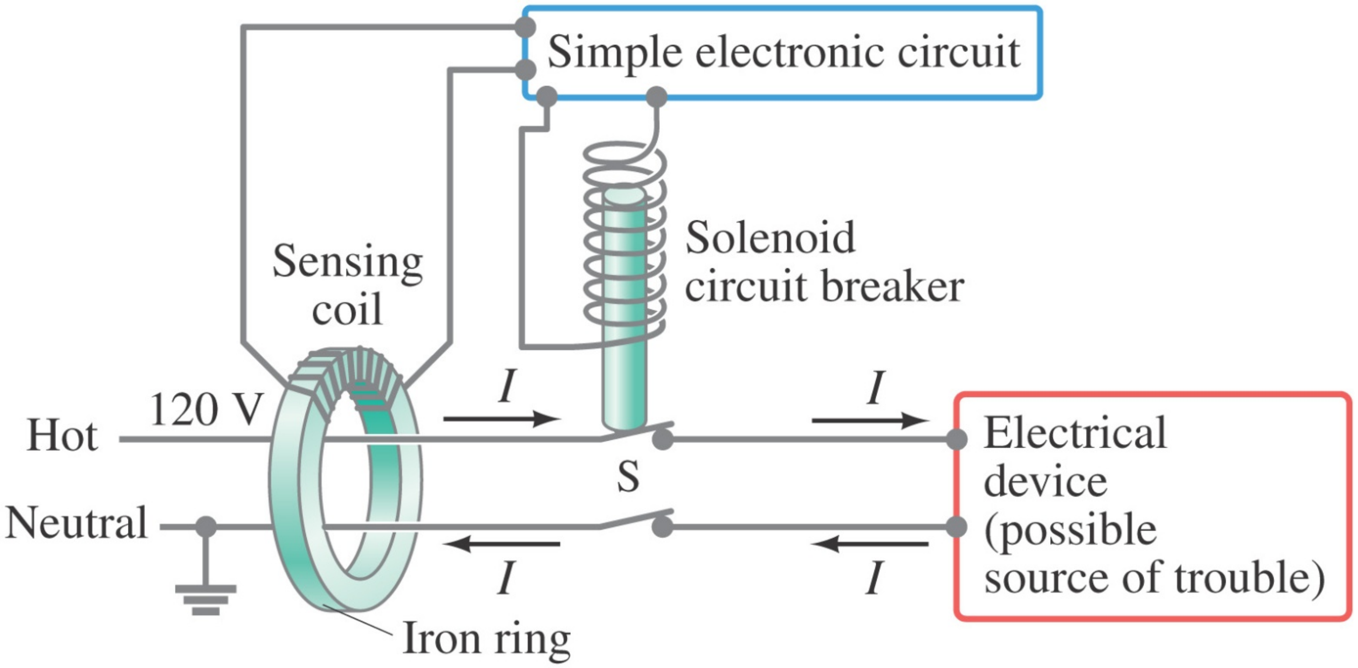
21.8 Applications of Induction: Sound Systems, Computer Memory, Seismograph, GFCI

A seismograph has a fixed coil and a magnet hung on a spring (or vice versa), and records the current induced when the earth shakes.



21.8 Applications of Induction: Sound Systems, Computer Memory, Seismograph, GFCI

A ground fault circuit interrupter (GFCI) will interrupt the current to a circuit that has shorted out in a very short time, preventing electrocution.



21.9 Inductance

Mutual inductance: a changing current in one coil will induce a current in a second coil.

$$\mathcal{E}_2 = -M\frac{\Delta I_1}{\Delta t} \qquad \textbf{(21-8a)}$$

And vice versa; note that the constant M, known as the mutual inductance, is the same:

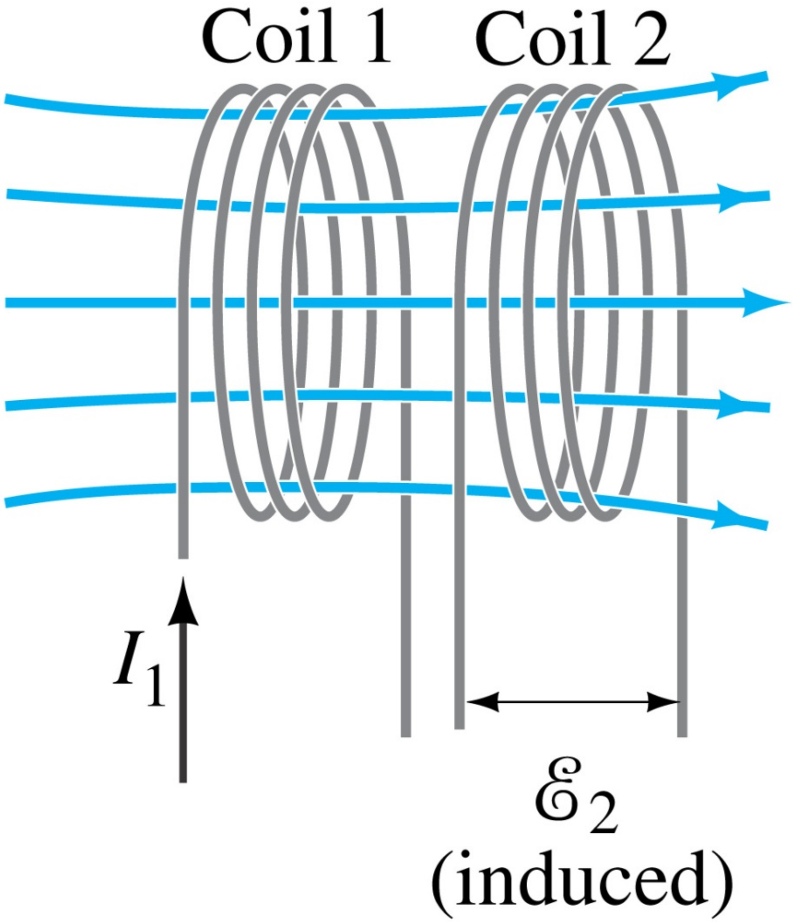
$$\mathcal{E}_1 = -M\frac{\Delta I_2}{\Delta t} \qquad \textbf{(21-8b)}$$

21.9 Inductance

Unit of inductance: the henry, H.

$1\ \mathrm{H} = 1\ \mathrm{V{\cdot}s/A} = 1\ \Omega{\cdot}\mathrm{s}.$

A transformer is an example of mutual inductance.



21.9 Inductance

A changing current in a coil will also induce an emf in itself:

$$\mathscr{E} = -L\frac{\Delta I}{\Delta t} \qquad (21\text{-}9)$$

Here, L is called the self-inductance.

21.10 Energy Stored in a Magnetic Field

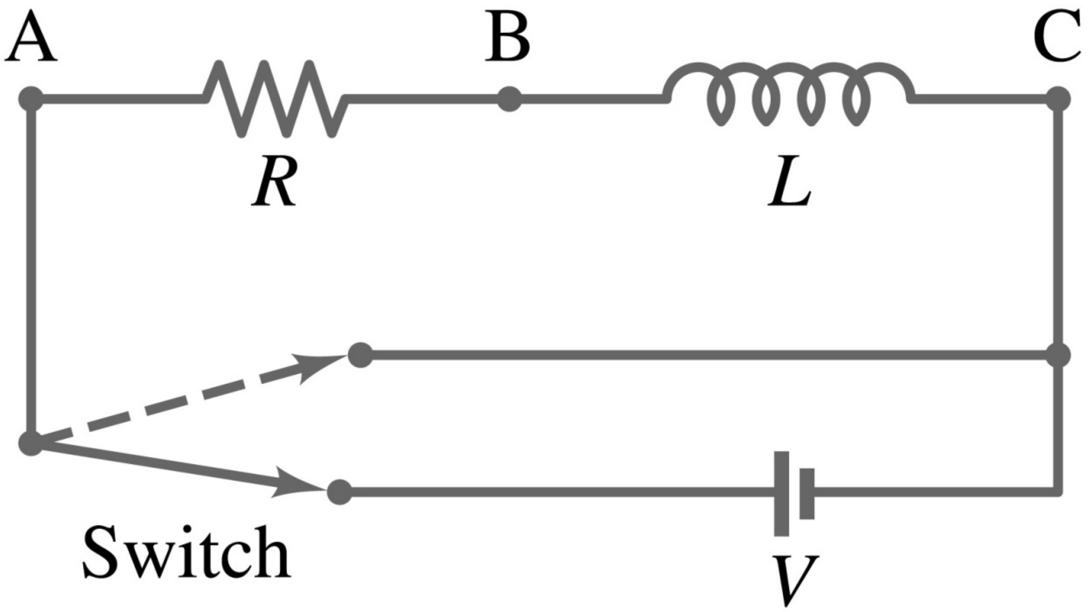
Just as we saw that energy can be stored in an electric field, energy can be stored in a magnetic field as well, in an inductor, for example.

Analysis shows that the energy density of the field is given by:

$$u = \text{energy density} = \frac{1}{2}\frac{B^2}{\mu_0} \qquad \text{(21-10)}$$

21.11 LR Circuit

A circuit consisting of an inductor and a resistor will begin with most of the voltage drop across the inductor, as the current is changing rapidly. With time, the current will increase less and less, until all the voltage is across the resistor.



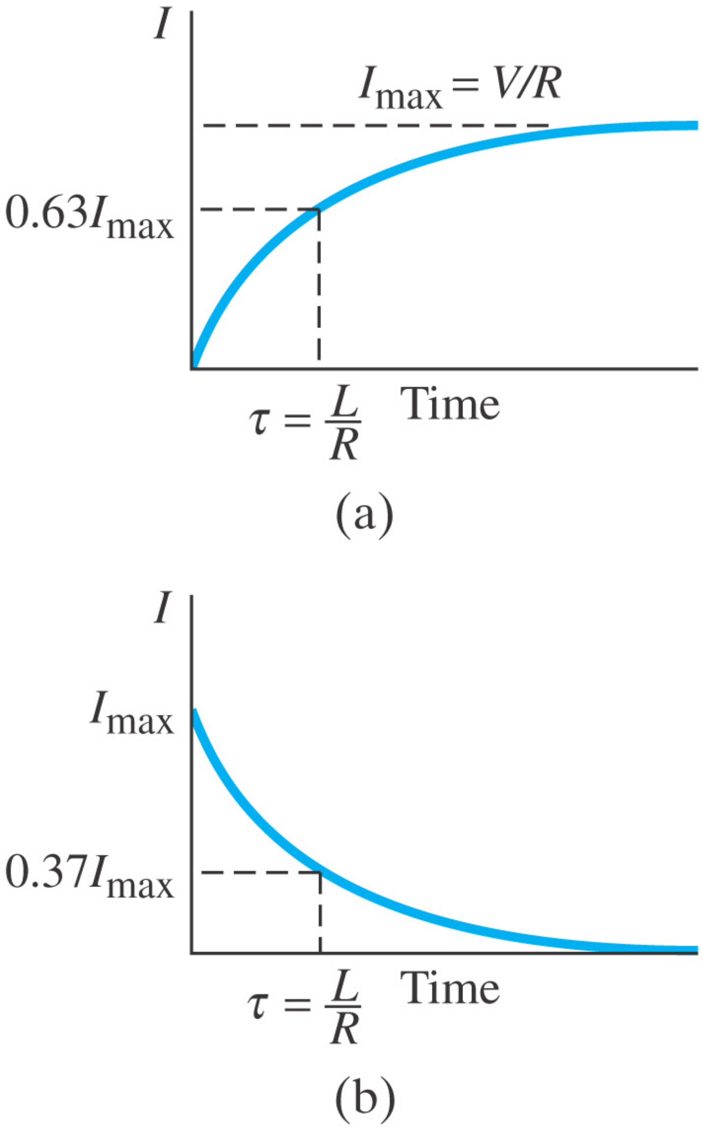
21.11 LR Circuit

If the circuit is then shorted across the battery, the current gradually decay away.

$$I = \left(\frac{V}{R}\right)\left(1 - e^{-t/\tau}\right)$$

$$I = I_{\max} e^{-t/\tau}$$

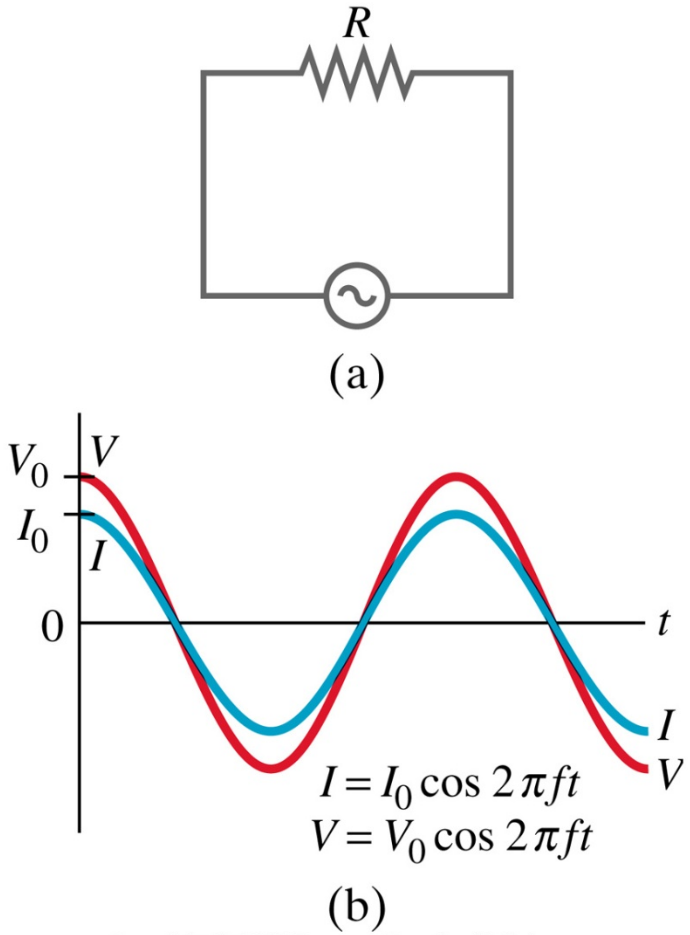
where $\tau = L/R$



21.12 AC Circuits and Reactance

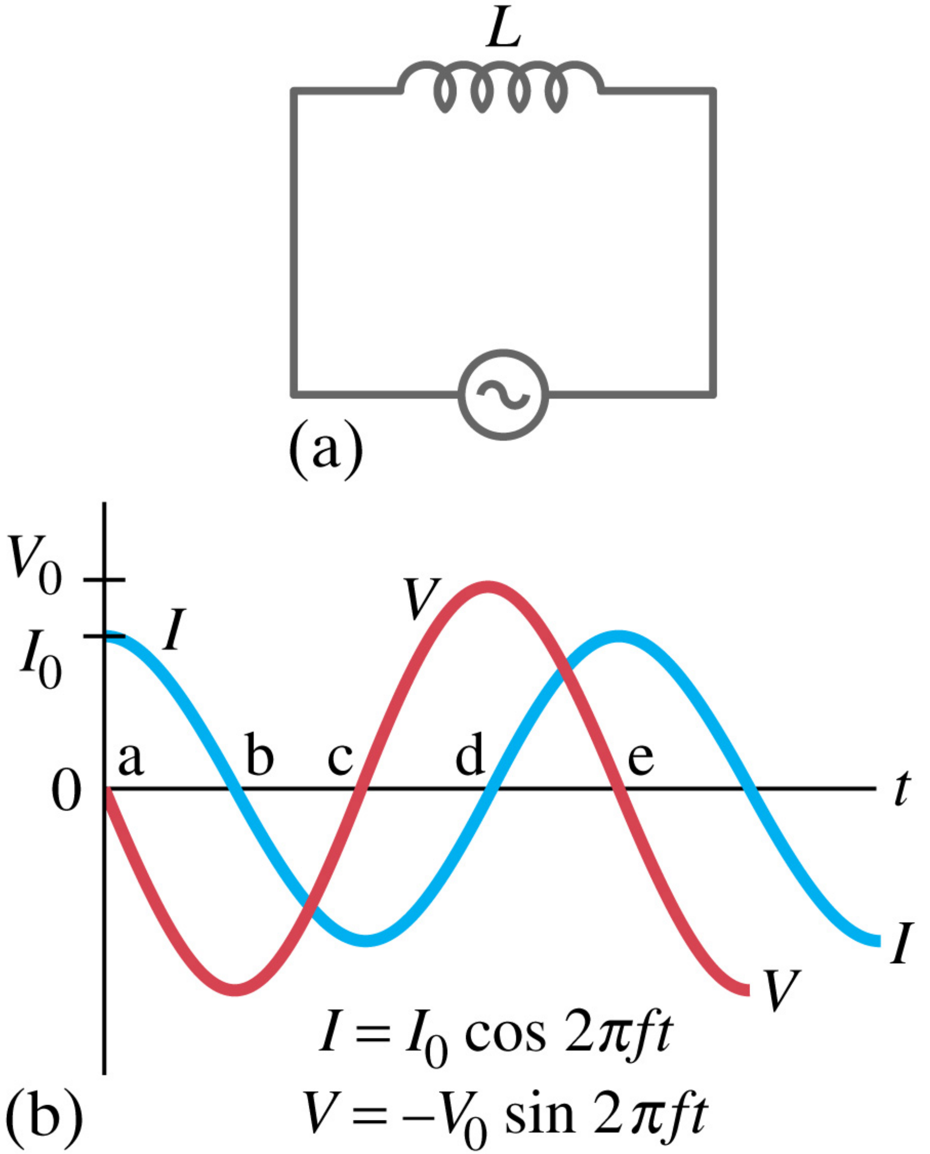
Resistors, capacitors, and inductors have different phase relationships between current and voltage when placed in an ac circuit.

The current through a resistor is in phase with the voltage.



21.12 AC Circuits and Reactance

The current through an inductor lags the voltage by 90°.

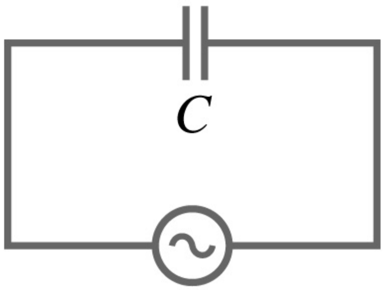


$$I = I_0 \cos 2\pi f t$$

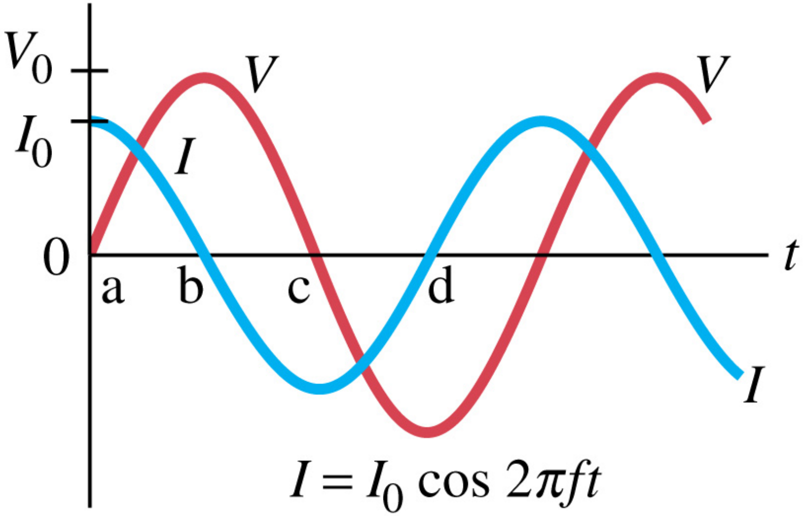
$$V = -V_0 \sin 2\pi f t$$

21.12 AC Circuits and Reactance

In a capacitor, the current leads the voltage by 90°.



(a)



$$I = I_0 \cos 2\pi f t$$

$$V = V_0 \sin 2\pi f t$$

(b)

21.12 AC Circuits and Reactance

Both the inductor and capacitor have an effective resistance (ratio of voltage to current), called the reactance.

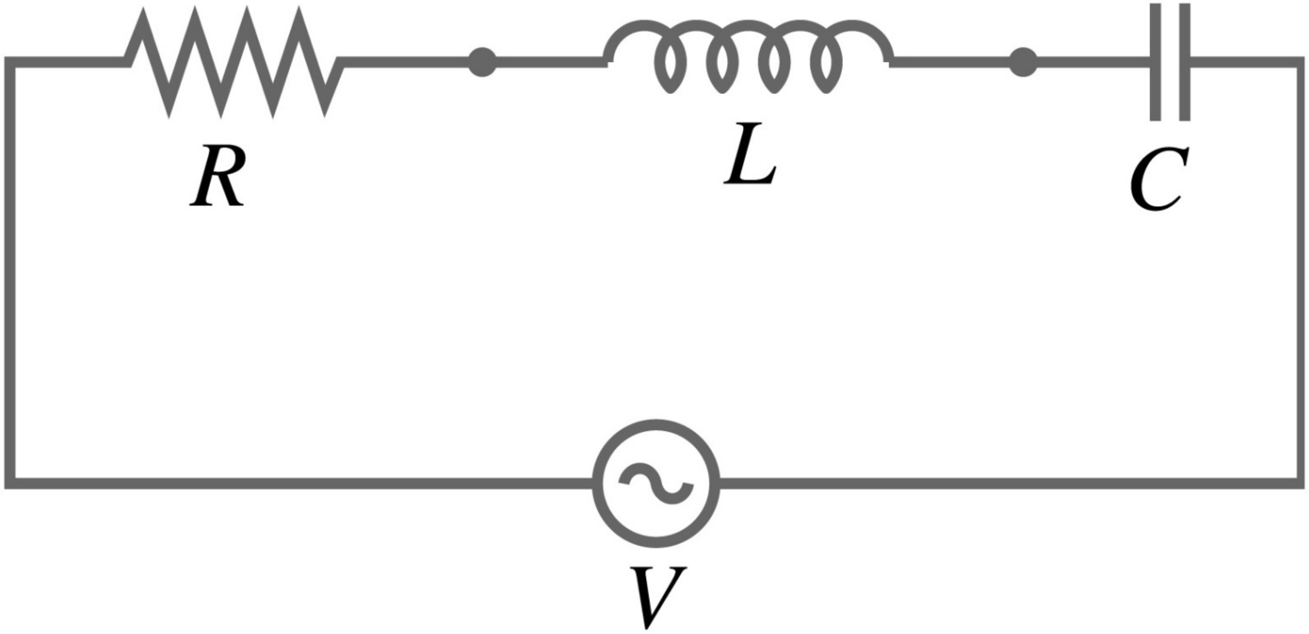
Inductor: $X_L = \omega L = 2\pi f L$ **(21-11b)**

Capacitor: $X_C = \dfrac{1}{\omega C} = \dfrac{1}{2\pi f C}$ **(21-12b)**

Note that both depend on frequency.

21.13 LRC Series AC Circuit

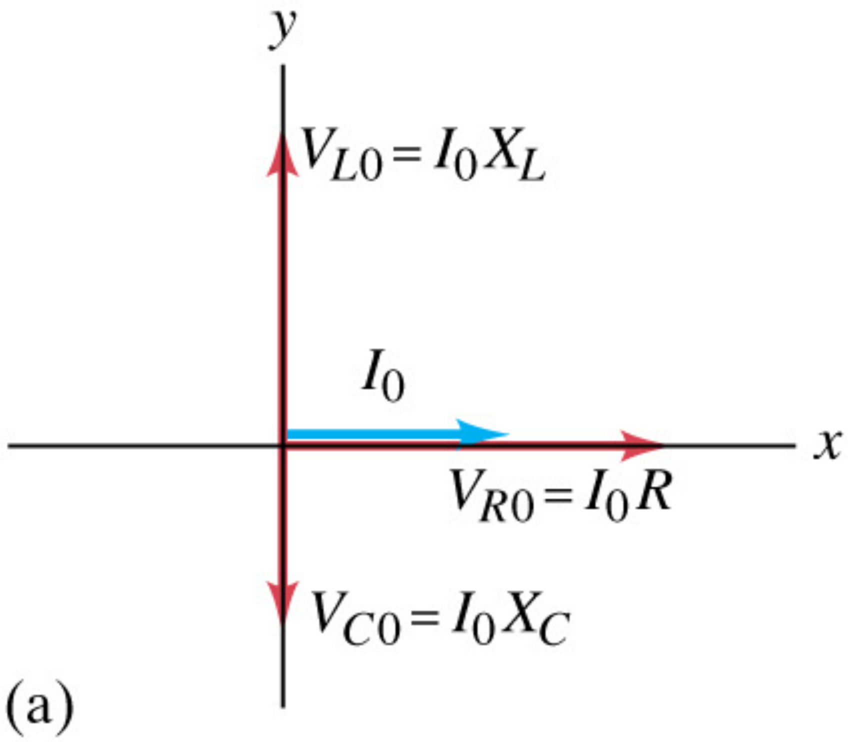
Analyzing the LRC series AC circuit is complicated, as the voltages are not in phase – this means we cannot simply add them. Furthermore, the reactances depend on the frequency.



21.13 LRC Series AC Circuit

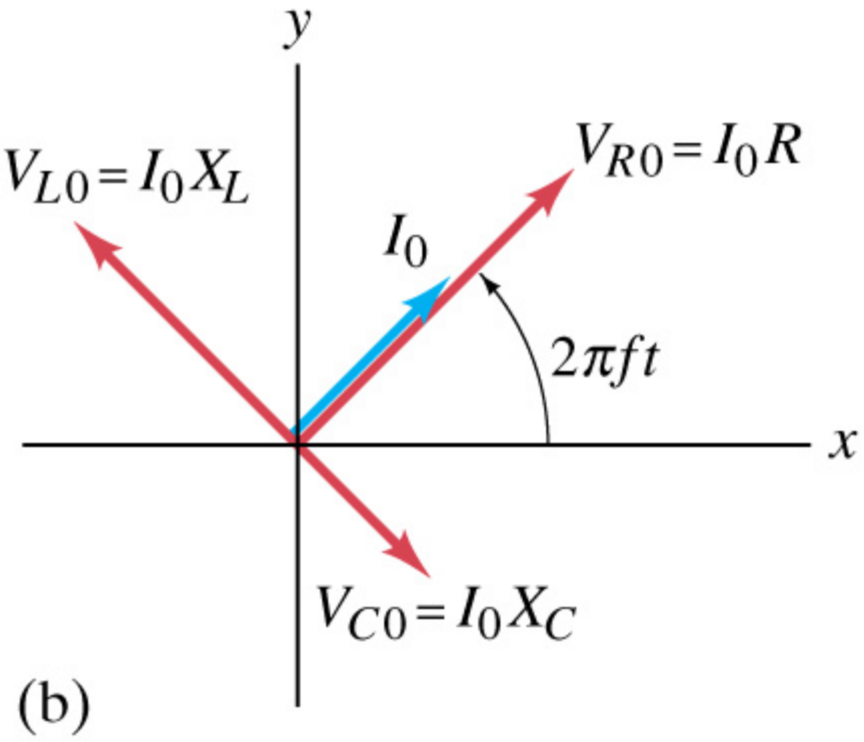
We calculate the voltage (and current) using what are called phasors – these are vectors representing the individual voltages.

Here, at $t = 0$, the current and voltage are both at a maximum. As time goes on, the phasors will rotate counterclockwise.



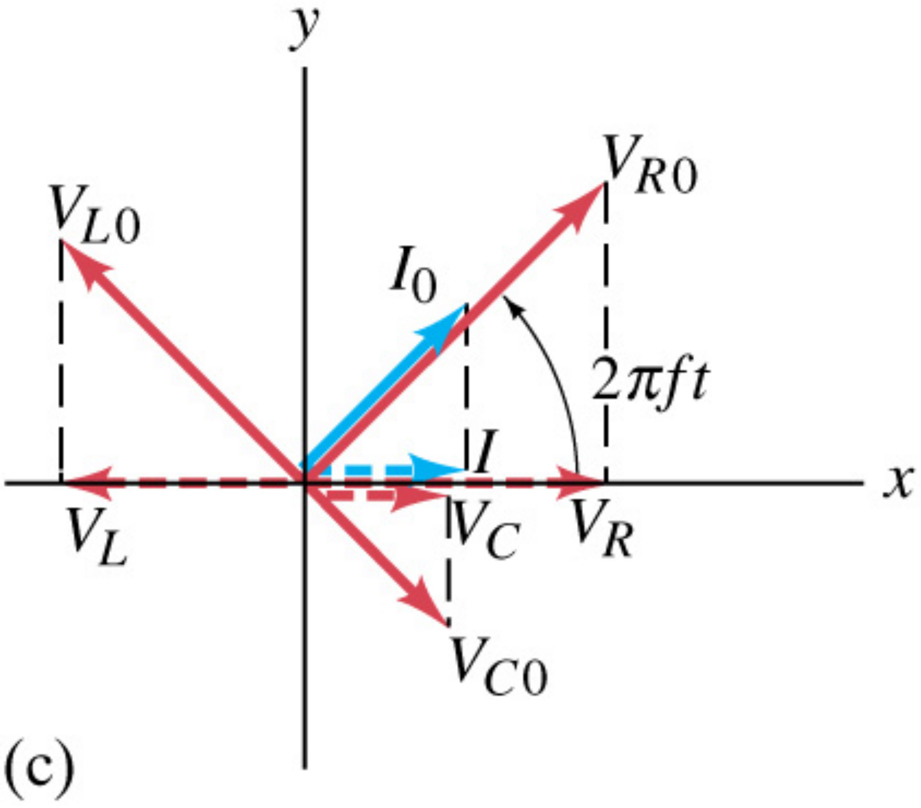
21.13 LRC Series AC Circuit

Some time *t* later, the phasors have rotated.



21.13 LRC Series AC Circuit

The voltages across each device are given by the x-component of each, and the current by its x-component. The current is the same throughout the circuit.



21.13 LRC Series AC Circuit

We find from the ratio of voltage to current that the effective resistance, called the impedance, of the circuit is given by:

$$Z = \sqrt{R^2 + (X_L - X_C)^2} \qquad \text{(21-15)}$$

21.14 Resonance in AC Circuits

The rms current in an ac circuit is:

$$I_{\mathrm{rms}} = \frac{V_{\mathrm{rms}}}{Z} = \frac{V_{\mathrm{rms}}}{\sqrt{R^2 + \left(2\pi f L - \frac{1}{2\pi f C}\right)^2}} \qquad \textbf{(21-18)}$$

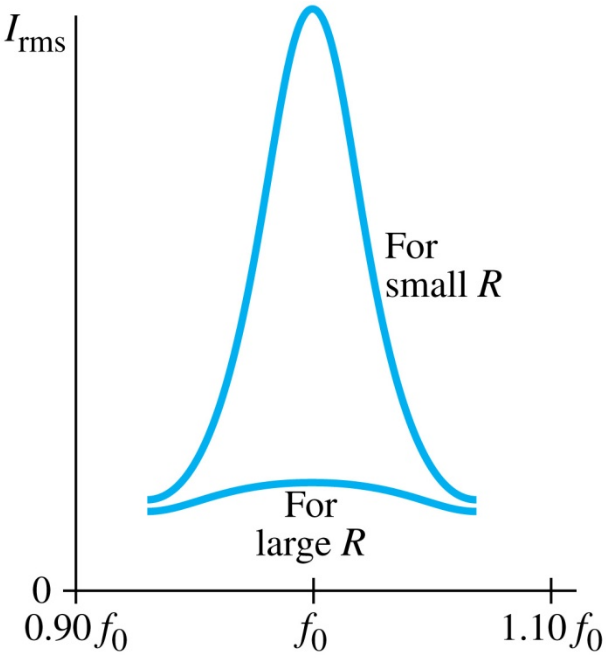
Clearly, I_{rms} depends on the frequency.

21.14 Resonance in AC Circuits

We see that I_{rms} will be a maximum when $X_C = X_L$; the frequency at which this occurs is

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC}} \qquad \textbf{(21-19)}$$

This is called the resonant frequency.



Copyright © 2005 Pearson Prentice Hall, Inc.

Summary of Chapter 21

- Magnetic flux:

$$\Phi_B = B_\perp A = BA\cos\theta$$

- Changing magnetic flux induces emf:

$$\mathscr{E} = -N\frac{\Delta\Phi_B}{\Delta t}$$

- Induced emf produces current that opposes original flux change

Summary of Chapter 21

- Changing magnetic field produces an electric field
- Electric generator changes mechanical energy to electrical energy; electric motor does the opposite
- Transformer uses induction to change voltage:

$$\frac{V_S}{V_P} = \frac{N_S}{N_P}$$

Summary of Chapter 21

- **Mutual inductance:**

$$\mathscr{E}_2 = -M\frac{\Delta I_1}{\Delta t}$$

- **Energy density stored in magnetic field:**

$$u = \text{energy density} = \frac{1}{2}\frac{B^2}{\mu_0}$$

- **LRC series circuit:**

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

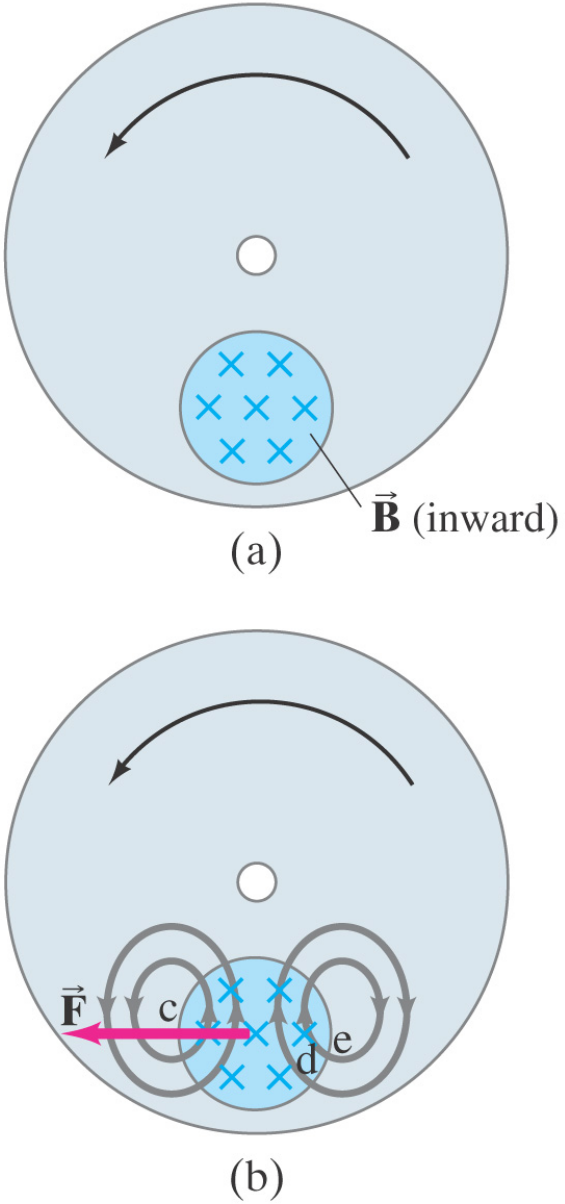
21.2 Faraday’s Law of Induction; Lenz’s Law

Problem Solving: Lenz’s Law

1. Determine whether the magnetic flux is increasing, decreasing, or unchanged.
2. The magnetic field due to the induced current points in the opposite direction to the original field if the flux is increasing; in the same direction if it is decreasing; and is zero if the flux is not changing.
3. Use the right-hand rule to determine the direction of the current.
4. Remember that the external field and the field due to the induced current are different.

21.6 Back EMF and Counter Torque; Eddy Currents

Induced currents can flow in bulk material as well as through wires. These are called eddy currents, and can dramatically slow a conductor moving into or out of a magnetic field.



Copyright © 2005 Pearson Prentice Hall, Inc.

Units of Chapter 21

- Induced EMF
- Faraday’s Law of Induction; Lenz’s Law
- EMF Induced in a Moving Conductor
- Changing Magnetic Flux Produces an Electric Field
- Electric Generators
- Back EMF and Counter Torque; Eddy Currents
- Transformers and Transmission of Power

Units of Chapter 21

- Applications of Induction: Sound Systems, Computer Memory, Seismograph, GFCI
- Inductance
- Energy Stored in a Magnetic Field
- LR Circuit
- AC Circuits and Reactance
- LRC Series AC Circuit
- Resonance in AC Circuits